On the Implications of and Search for "Large" Extra Dimensions

Fred Niell

5/4/01

Physics 523, FT2, Final Paper

Introduction

New theories have recently proposed a solution to the unification hierarchy problem as well as a framework for a self-consistent quantum theory of gravity. By allowing the gravitational and gauge interactions to unite on an electroweak scale, we can attribute the relative weakness of gravity at long distances to the existence of large extra dimensions. These extra dimensions must occupy a space whose size must be roughly comparable to the unification energy scale. These theories predict and explain exotic phenomena from Newton's gravitational force law changing functional form at short distances to accelerator data exhibiting large $\not{E}_T$ due to gravitons escaping into the bulk. Some theories even hint at the chance of a grand unification theory (GUT) through a treatment of gravity in a large extra dimensional framework. An interesting chronology has arisen in the last two or three years surrounding this search. In this paper, I will attempt to review some of the more interesting aspects of this theory's implications and possibilities. A general review of the Standard Model problems and these theories' proposed answers and some of the experimental evidence for and against the extra dimensional view will be presented.

Unification of Electroweak forces and Gravity

Unification

The holy grail of a grand unification theory (GUT) has eluded physicists since the first attempt to unite the strong, weak, gravitational, and electromagnetic forces. While no particular GUT seems particularly likely as an explanation of nature, several have been shown to be consistent with experimental data. Most of the theories have concerned themselves with unifying quarks and leptons with electroweak and strong forces. For leptons and quarks, the natural scheme was to put the particles into a larger symmetry group. A simple scheme involves an $SU(5)$ representation, which without inputs to the theory, gives fractional quantization of quark and even the neutrality of atoms. Typical GUTs unifying electroweak and strong forces attempt to calculate an energy scale at which the coupling strengths of the forces are (hopefully) linearly related.

Unfortunately, none of these theories include any gravitational interactions. There is a good reason; gravity's mass scale is huge, as the length scale associated with gravity is the Planck length, $R_{Pl} = 10^{-33}$cm. That means that the current tools for calculating unification theories cannot bring gravity and the electroweak forces together; the coupling strengths would not converge to the same place in coupling-mass scale space [11].

Quantum field theory has shown us that from first principles, one can construct a lagrangian utilizing various gauge groups that will behave like nature. The strength of modern interpretations of field theory use a covariant derivative lagrangian,

$$\mathcal{L}_{fermion} = \sum_f \bar{f} i \gamma^\mu \mathcal{D}_\mu f \tag{1}$$

with the derivative $\mathcal{D}_\mu$ being

$$\mathcal{D}_\mu = \partial_\mu - i g_1 \frac{Y}{2} B_\mu - i g_2 \frac{\tau^i}{2} W^i_\mu - i g_3 \frac{\lambda^a}{2} G^a_\mu \tag{2}$$

This lagrangian gives us a description of nature that we have come to believe for the most part unequivocally in the field of particle physics. The second term in $\mathcal{L}$ is the $U(1)$ symmetry piece, due to electromagnetic interactions and the photon. The third term is the $SU(2)$ group piece, giving us charged and neutral current interactions, W^{+-0}, Z bosons. The last term is the $SU(3)$ group term, giving us the quark gluon interactions. The $g_{1,2,3}$ factors in front of the terms in the covariant derivative are the coupling strengths of the various interactions. In group theory terminology, we call this formalism $SU(3) \times SU(2) \times U(1)$ gauge theory [12, 11].

We know that the electromagnetic coupling g_1 decreases at the high energy scale. Since g_2 and g_3 are asymptotically free, we can let them increase as the energy scale increases. At some point, the g_2 and g_3 slopes could cross and g_1 might be made to meet them. At least, that is the idea behind unification. If we further let the $SU(3) \times SU(2) \times U(1)$ be subgroups of the larger $SU(5)$ symmetry group whose symmetry is spontaneously broken at high energy, we have what is generally called grand unification.

So the couplings, g_i are energy dependent. Typically, one uses the fine structure running coupling "constant" when talking about unification;

$$\alpha_1 = \frac{g_1^2}{4\pi} \approx \frac{1}{100}$$
$$\alpha_2 = \frac{g_2^2}{4\pi} \approx \frac{1}{30}$$
$$\alpha_3 = \frac{g_3^2}{4\pi} \approx \frac{1}{3 \rightarrow 10}.$$

Since the αs are anything but constant, we have to calculate them for different energies, including however many loop corrections we can. Since there is a large amount of data available, we have a fairly well accepted value of $\alpha = 1/137$. The asymptotic behavior of $\alpha_{2,3}$ gives us the ability to make the strong force strong and the weak forces what the are, while at some large energy making them roughly equal. What we want to have is one overriding α_5 for the large symmetry group which at the unification energy, $M_{unification}$, is equal to the other αs in the following manner,

$$\alpha_5 = \frac{5}{3}\alpha_1(M^2_{unification}) = \alpha_2(M^2_{unification}) = \alpha_3(M^2_{unification}) \tag{3}$$

Following Kane [11], we can use the two asymptotically free $\alpha_{2,3}$ and nail down the $M_{unification}$ using a known mass. Based on the gauge couplings and the observed form of the interactions of the QED and QCD terms, we can write

$$\log \frac{M_{unification}}{M_{known\ data}} = \frac{6\pi}{11}\left(\frac{1}{\alpha_2(M_{known\ data})} - \frac{1}{\alpha_3(M_{known\ data})}\right) \tag{4}$$

If we choose the mass of the W boson as $M_{known\ data}$ and $\alpha_2 = 1/30$ and $\alpha_3 = 0.11$ (best values from experiment), we get

$$\log \frac{M_{unification}}{M_W} \approx 35.83$$

$$M_{unification} \approx 10^{17} GeV$$

Since the dependence of the unification energy is logarithmic, choice of α data is crucial. Since the Planck scale is $M_{Pl} \sim 10^{18}$Gev, this is not so far away from the scale at which the other gauge forces come together, and it was believed that careful study would reveal the unification energy of the other gauge forces is actually closer to the Planck scale. However, newer data showed trends in the opposite direction, pushing them further away. Some new theory was needed if one was to unite gravity and the other gauge forces [11].

Appealing to available data, there appear to be two fundamental mass scales in nature, the electroweak scale and the Planck scale. Explaining this hierarchy has been a goal of many of the theories beyond the standard model (SM) for the last thirty years. Effective field theories have been constructed around the electroweak mass scale that explain the origin of the hierarchy. However, the only quantum gravity theories are built around R_{Pl}. The rudiments of string theory inspired the search for extra dimensions.

"Curling" up extra dimensions, model building

One important motivation of the extra dimensional search is the thought that the electroweak mass scale is the only small mass scale. The proposition is that M_{Pl} is not a fundamental scale at all, rather gravity is weak due to the size R of the extra dimensions relative to the weak scale. To construct a model, we allow the m_{EW} to be the ultraviolet cutoff of our theory. From there, we say there are n compact extra dimensions, whose characteristic radius (size) is R. These dimensions are typically said to be "curled" up, ala Kaluza Klein compactification. The resultant topology is a compact n dimensional manifold (M_n) along with our familiar R^4; [$R^4 \times M_n, \; \forall \; n \geq 2$]. Several papers suggest possible topologies that have more or less favorable characteristics for calculations. As discussed below, we can set our Planck scale in the $(4+n)$ picture $\sim m_{EW}$. Since SM gauge forces have been probed extensively to the range of 100 GeV, we must assume that the SM particles cannot propagate through the bulk $(4+n)$ dimensions. We must let the SM particles propagate on a 4-dimensional submanifold of the $(4+n)$ bulk. There would only be one field allowed to span the $(4+n)$ dimensions; the graviton. The graviton would have to span all dimensions, as it is the only gauge particle that "lives" in all the dimensions. [1, 7, 8]

Ramifications: Escape, gravitons, protons, and accelerators

Several exciting consequences arise from this framework. First off, our gravitational strength becomes comparable to other gauge forces at the electroweak mass scale, $m_{EW} \sim 1$ TeV. In light of the new collider projects in the near future, NLC, LHC, etc., we can expect an elucidation of electroweak symmetry breaking. This theory posits that we would also be probing the nature of quantum gravity at the same time.

Second, for the case of 2 extra dimensions, $(n = 2)$, the gravitational force law should change from $\frac{1}{r^2}$ at distances greater than R to $\frac{1}{r^4}$ for distances on the order of R (a subject of later discussion). New accelerators are also expected to see new signatures in their detectors. $\not{E}_T$ signatures in detectors are typical, but there is a new series of propositions for the unidentified interactions. Fermion pair production including a produced graviton would lead to large $\not{E}_T$ and jets, in a particular sector with low backgrounds [$e^+e^- \rightarrow f\bar{f}$G]. Beam particles that radiate gravitons through a gravitational interaction before they actually collide would carry away large amounts of energy just before the collision occurred. This graviton radiation has been given the cute name "gravistrahlung." The new linear collider projects, with $\sqrt{s} \approx 500$ GeV, will be perfect instruments to allow us to study quantum gravity from 1 TeV to 4 TeV. $M_{unification}$ could be studied at a e^+e^- linear collider by looking at the proposed KK graviton emission associated with two fermions. Éboli et. al. demonstrated that the crossection σ for $e^+e^- \rightarrow f\bar{f}$G depends on the number of extra dimensions in your model. They calculated a table of possible cross sections for different choices of n dimensions, taking into account hadronic, muonic, and combined decay and production channels, and their statistical significance. Their data showed that for the simplistic cases of $n = 2$ (the most studied limit to date, due to ease of calculation), a linear collider with $\sqrt{s} = 500$ GeV and integrated luminosity $\mathcal{L} = 500 fb^{-1}$ will give 5σ limits on the $M_{unification}$ all the way up to 4.1 TeV, and smaller for other combinations of $\mathcal{L}$ and n [6].

A more exotic mechanism for large $\not{E}_T$ involves the proposed escape energy, E_{esc}. Collisions of $\sqrt{s} \geq m_{EW}$ would have the ability to give SM fields momentum in the extra dimensions, carrying away energy, and in fact escaping our 4-d world. In fact, phase space arguments alone favor the escape above all else at energies greater than the E_{esc}. This leads to the idea of a sharp cutoff to the transverse momentum that we can detect in our 4-dimensional world, at $\sqrt{s}$ or $p_T = E_{esc}$. This "energy maximum" could easily be seen at NLC, LHC, or other new machines, if the beams are high enough energy to have $\sqrt{s} \geq E_{esc}$. Above such an energy maximum, the usual particles would simply cease to be produced! [6, 1]

Extensions to the SM must contend with certain things that *cannot* be violated, such as the decay lifetime of the proton. When constructing a Lagrangian in more than 4 dimensions, one must make sure that higher dimension operators that give rise to proton decay are adequately suppressed; at least to the m_{EW} scale in our case. Often the “problem” terms in the lagrangians are baryon number violating gauge bosons that arise from imbedding the $SU(3) \times SU(2) \times U(1)$ in the larger $SU(5)$. If quarks are cast as $SU(2)$ doublets, gauge bosons $W^{\pm}$ act as “raising” and “lowering” operators, giving $u \rightarrow d$ transitions, as well as others. Likewise, putting leptons and quarks together into a $SU(5)$ multiplet gives rise to the typical gauge bosons $(\gamma, W^{\pm}, Z, g)$, but new bosons can arise, allowing quark-lepton exchange. These $SU(5)$ models can allow a new gauge particle (for example X) whose couplings violate baryon number. Integrating the particles out gives rise to protonic decay via $p \rightarrow \gamma\ e^{+}$ or $p \rightarrow \pi$, via a new gauge particle $\bar{X}$, Figure(1), [11, 12]. This interaction is best written using the helicity states of the quark and electron fields. If we let u_L, d_L, u_R, and e_R be two component spinors, we can write down an effective interaction lagrangian,

$$\Delta\mathcal{L} = \frac{2}{m_X^2}\epsilon_{abc}\epsilon^{\alpha\beta}\epsilon^{\gamma\delta}e_{R\alpha}u_{R\alpha}u_{Lb\gamma}d_{Lc\delta} \tag{5}$$

The X boson now can allow lepton $\Longleftrightarrow$ quark exchange, which would give us a baryon number violation. The typical mass estimated for these bosons would be 10^{16} GeV. The typical lifetime of the proton under this interaction is 10^{31} years, and experiments ongoing since 1983 have searched for proton decay. The experiments to date have ruled out protonic decay to lifetimes of $\sim 10^{31} - 10^{33}$ years, depending on decay mode, but further experimentation will certainly show protonic decay to be false. So our models shouldn't allow proton decay.

One can fix this nasty problem by invoking certain symmetries in the lagrangian $\mathcal{L}$. By imposing R parity and utilizing MSSM tools to suppress baryon number violating terms, one can shut off proton decay in low-energy supersymmetric models. A well studied example [7] uses a "wall" visualization tool. If we allow the SM fields, fermions and quarks, to propagate freely through the three space dimensions but be localized, or fixed, on a 2 dimensional wall (the remaining 2 dimensions in a $(4+n) = 6,\ n = 2$ model). Of course this "wall" is a fictitious construct, but a useful visualization tool. The "thickness" of said wall would be the E_{esc} mentioned earlier. To suppress proton decay, one needs to keep the interaction of fermions with other fermions at least exponentially suppressed, as well as quarks. Arkani-Hamed and Schmaltz [7] went to considerable effort to construct a wave function that would span the 5 spatial dimensions in this framework. Their calculations gave wave functions whose representation in the "extra" two dimensions happens to be a gaussian. We are the presented with the idea of a wall with gaussian packets fixed on the surface. Clearly, localization of the SM fields in this manner gives rise to the required non-symmetry violating interaction on the wall. And thus, the proton is safe from decay [4, 7].

Gravity

New potential for gravity, no pun intended

Until recently, Newton's inverse-square law was believed to hold for all distances greater than the Planck length, $R_{Pl} = \sqrt{G\hbar/c^3} = 1.6 \times 10^{-33}$cm. The Kaluza Klein compactification of dimensions has raised the question of unification of gravity and electroweak forces around the 1 TeV energy scale. If we assume there are $(4+n)$ dimensions, we must rescale the $1/M_{Pl}$

gravitational strength. We set the new $M_{Pl} \sim m_{EW}$. Thus the weak and gravitational scales are neatly combined, and luckily form the ultraviolet cutoff of the theory.

We are left with n compact extra dimensions in which gravity is allowed to propagate, the "normal" gauge fields only localized to $\frac{1}{m_{EW}}$ within them. Below the characteristic size of these extra dimensions, gravity should have a different potential by a continuation of Gauss's law into $(4+n)$ dimensions. As before, R is our characteristic radius of the extra dimensions. Using the usual notation for gravitational interactions, test masses m_1, m_2 at a distance r, we can construct a new effective, rescaled gravitational potential.

However, the gravitational potential will depend on how we "curl" the extra dimensions up. If we compactify the n extra dimensions in circles each with radius R, we have a toroidal sub-manifold. The gravitational potential should obey Laplace's equation in $(n+3)$ dimensions, giving us

$$V_{(n+4)} = -\sum_{\vec{m}} \frac{G_{n+4} M}{\left(r^2 + \sum_{\mathrm{i}=1}^{\mathrm{n}} (\mathrm{x_i} - 2\pi \mathrm{R_i m_i})^2\right)^{\frac{(\mathrm{n}+1)}{2}}}. \tag{6}$$

This is simply the vector distance between the two points in $(4+n)$ dimensions, with the extra dimensions having a "looping" symmetry, mod 2π. This looping allows us, at large R_i's to drop all terms except the one the leaves us with Newton's law. Other schemes for "curling" the extra dimensions on other geometrical manifolds can lead to different metrics, giving different forms for the gravitational potential. Spherical and other geometries give particularly nasty forms, involving techically difficult calculations for the potential [13].

Since the easiest to calculate manifold geometry is that of a toroid, it has been the most studied geometry to date [1, 3]. Particularly, Arkani-Hamed, et. al. studied this geometry for their paper. This geometry gives us two nice looking potentials, one for the close interactions, one for the far interactions;

$$V(r) \sim \frac{m_1 m_2}{M_{Pl(4+n)}^{n+2}} \frac{1}{r^{n+1}}, (r \ll R) \tag{7}$$

and

$$V(r) \sim \frac{m_1 m_2}{M_{Pl(4+n)}^{n+2} R^n} \frac{1}{r}, (r \gg R) \tag{8}$$

Notice that if the masses are at $r \gg R$, we get back the usual inverse-square law force. To get our new effective M_{Pl} for our $(4+n)$ dimensions, so we need to examine the strength of the gravitational pull; simply,

$$M_{Pl} \sim \sqrt{M_{Pl(4+n)}^{(2+n)} R^n}. \tag{9}$$

We then set the m_{EW} to the new effective M_{Pl} to calculate the new "size" of the dimension R,

$$R = \frac{\hbar c}{m_{EW}} \left(\frac{M_{Pl}}{m_{EW}} \right)^{\frac{2}{n}}. \tag{10}$$

Which for our purposes is easier written:

$$R \sim 10^{\frac{30}{n} - 17} \times \left(\frac{1TeV}{m_{EW}} \right)^{1+\frac{2}{n}} \tag{11}$$

Armed with a tool to calculate the relative size of our new dimension, we can examine the ramifications of the two simplest cases; $n = 1, n = 2$ [1, 2].

Dimensional restrictions

For $n = 1$ $R \approx 10^{13}$ cm, which corresponds to solar-system distances. This is of course not possible, as we see gravity's inverse square law stable out past our own solar system- thus we can exclude this number of dimensions on a simply empirical basis. However, since the characteristic distance R is dependent on n in the exponent, raising up to $n \geq 2$ causes large changes in our characteristic distance. The particularly exciting result is the possibility of $n = 2$. For $n = 2$, we have ($R \approx 100\mu$m $-$ 1mm). Gravity up until very recently had not been measured to any precision beyond a few millimeters, and theorists felt that there might be a possibility of some wild, new physics at this smaller scale. Other interesting phenomena would arise at short length scales. For example, were supersymmetry broken at short distances, Yukawa interaction generating scalar particles would produce mm-scale effects. Recent determinations [9, 10] of the cosmological constant based on supernovae support a length scale of $(\hbar c/\Lambda)^{\frac{1}{4}} \approx 0.1$mm. [1, 2, 7, 3]

An interesting problem arises, though. SM gauge forces have been measured to far below the few millimeter scale (beyond 100 GeV), and if there was to be any new physics below this "barrier," the SM particles must not be able to propagate in these new dimensions, just localized to a 4-dimensional sub-manifold (a 3-brane, in modern parlance, plus time) of the $(4 + n)$ dimensions. So- the apparent "weakness" of gravity is taken up in these new dimensions; the only particle that propagates through the $(4 + n)$ bulk are the new gravitons. Everything we experience occupies the 4-dimensional world, while gravity has an extra n dimensions. Clearly one might want to measure gravity below this length scale and look for some departure from Newton's inverse-square law force. [2]

Tests of gravity on small scale

Prior to November 2000, the greatest precision to which gravity's inverse-square law had been tested was greater than 1 mm. Since there was so much interest in the possibilities and ramifications of gravitational law deviation, Hoyle, et. al [3] constructed a high precision torsion pendulum to measure the gravitational constant. The apparatus included a 1-meter long fiber with a disk at the end, whose period of torsional rotation was coupled to the gravitational attraction of the disk to another mass. By varying the distance between the disk and the mass they could see the inverse square law effect in the measured rotational periods. Through a significant amount of data reduction, Hoyle et. al. were able to show that Newtonian physics holds down to 218μm with their apparatus.

The new data constrain the size and unification energy scales significantly, but do not entirely discount the possibility of "large" extra dimensions. The new bound pushes the unification energy from $\approx$ 1 TeV to $\approx$ 3.5 TeV. A new experiment by the same collaboration is being designed to improve the precision of the experiment and hopefully allow for smaller separations of mass and disk- probing the inverse square law to even smaller distances. Of course, one strong goal of this gravitation research was the validation or contraindication of the large extra dimensional framework. The framework is useful in that it solves the hierarchy problem in a novel manner, yet the current gravity results are pushing the unification scale at least to 4 TeV. This is nearing a contraindication, as the 1 TeV electroweak mass scale is very well known through experiment. The main tenet of the theory was that the *only* fundamental mass scale is the 1 TeV electroweak scale. Further experiments are expected to show inverse square law consistency down past sub-micron range. This would push the unification mass so far away from the electroweak scale that the entire theory would have to be re-thought, as a new hierarchy would be introduced! [3]

Conclusion

Large extra dimensions are a very attractive, convenient solution to the hierarchy problem, giving glimmers of hope for a GUT. Several researchers have proposed theoretical frameworks that give us unification of electroweak and gravitational forces at a tantalizingly close and familiar energy scale. Perhaps because of the apparent facility and utility of the theory, much time has been put into its study and exploration. Such a theory has such broad sweeping consequences that its study is almost glamorous. Future experiments could make ground breaking findings from possible collider signatures at LHC and future linear colliders to table-top gravity experiments giving $\frac{1}{r^4}$ data.

However, the overriding sense one gets from reading the journals is that the likelihood of this theory actually coming to fruition is slim. The most recent, cutting edge gravity data push the unification mass scale farther from where we want it, creating a new hierarchy problem. Further data will almost certainly push it farther away, effectively pushing the theory in its current form out of the realm of possibility.

The pursuit of science is fraught with pitfalls and blind alleys. However, science cannot move forward without some lateral motion. How can we find the correct theory of the universe without thinking of a few that are in fact wrong? Since the consequences of this theory are so fascinating and promising, we should not discount this theory immediately, and I for one will certainly be looking at what the future holds for this innovative theory.

Figures

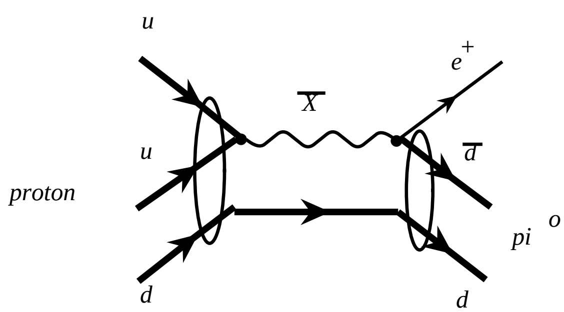

Figure 1: Decay diagram of proton to neutral pion via exchange of "new" $\bar{X}$ SU(5) gauge boson

References

[1] "The Hierarchy Problem and New Dimensions at a Millimeter", N. Arkani-Hamed et. al., Phys.Lett. B429 (1998) 263-272

[2] "Neutrino Masses from Large Extra Dimensions", N. Arkani-Hamed, et. al., arXiv:hep-ph/9811448

[3] "Sub-millimeter tests of the gravitational inverse-square law: a search for 'large' extra dimensions", C. Hoyle, et. al., Phys.Rev.Lett. 86 (2001) 1418-1421

[4] "New Dimensions at a Millimeter to a Fermi and Superstrings at a Tev", I. Antoniadis, et. al., Phys.Lett. B436 (1998) 257-263

[5] "Extra dimensions as an alternative to Higgs mechanism?", M. Shaposhnikov, P. Tinyakov, arXiv:hep-th/0102161

[6] "Direct Signals for Large Extra Dimensions in the Production of Fermion Pairs at Linear Colliders", O. Éboli, et. al., arXiv:hep-ph/0103053, Submitted to Phys. Rev. D

[7] "Heirarchies without Symmetries from Extra Dimensions", N. Arkani-Hamed, M. Schmaltz, Phys.Rev. D61 (2000) 005-033

[8] "Infinitely Large New Dimensions", N. Arkani-Hamed, et. al., Phys.Rev.Lett. 84 (2000) 586-589

[9] A.G. Rice, et. al., Astron. J. **116**, 1009 (1998).

[10] S. Perlmutter et. al., Astrophys. J. **517**, 565 (1999).

[11] "Modern Elementary Particle Physics: The Fundamental Particles and Forces", G. Kane, Addison-Wesley, 1993.

[12] "An Introduction to Quantum Field Theory", M. Peskin, D. Schroeder, Perseus, 1995.

[13] "Deviations from the $1/r^2$ Newton law due to Extra Dimensions", A. Kehagias, K. Sfetsos, Phys.Lett. B472 (2000), 039-044

LaTeX